Simulation intervals for real-time scheduling

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1. Uniprocessor systems
   1. State of the art
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2. Multiprocessor systems
   1. Counter-intuitive example
   2. State of the art and recent results

3. Summary of open problems
1. UNIPROCESSOR SYSTEMS
**Notations and assumptions**

- **Concrete**
- Depending on the results
  - Implicit/constrained/arbitrary deadlines
  - Independent (fully preemptive)/Precedence constrained/Resource constrained
Notations and assumptions

Scheduling algorithms

- Deterministic and memoryless (D&M)
  - Scheduling decision is based on the system’s state
    - Remaining work for each job
    - Local clocks \((t-O_i) \mod T_i\)
  - Fixed-task priorities (FTP) – like RM, DM, OPA
  - Fixed-job priorities (FJP) – like EDF
  - Any D&M scheduler – online, offline
1.1 State of the art

Seminal result [LM80]

Hyperperiod $H = \text{lcm}(T_i)$

$O^{\text{max}} = \max(O_i)$

Context:

- [LM80] FTP, independent, constrained deadlines
- [GD99] FJP, independent, arbitrary deadlines
- Upper bound for: FTP, indep, constr. [GD97]
- Upper bound for: any D&M, prec., mutual ex., constr. [CG04]
1.1 Specific FTP bound [GD97]

- Simulation interval
- \( s_1 = O_1 \)
- \( s_2 = \max \left( O_2, O_2 + \left\lfloor \frac{s_1 - O_2}{T_2} \right\rfloor T_2 \right) \)

Periodicity of highest priority task

- \( s_1 = O_1 \)
- \( T_1 \)
- \( O_2 \)
- \( T_2 \)
- \( O_2 + \left\lfloor \frac{s_1 - O_2}{T_2} \right\rfloor T_2 \)
- \( \text{lcm}(T_1, T_2) \)
1.1 Specific FTP bound /contd

\[
s_n = \max \left( O_n, O_n + \left\lfloor \frac{s_{n-1} - O_n}{T_n} \right\rfloor T_n \right) \leq O_{\text{max}} + H
\]

**Context:**
- FTP, indep., constr.
1.1 Any D&M [CG04]

- The first window of size H with exactly H(1-U) idle slots is the cycle
  - cannot start later than $O_{\text{max}} + H$
  - always follows an idle time or starts at 0

- Integer number of jobs in the transient phase
- Integer number of jobs in the steady (cyclic) phase

- Context:
  - Any D&M, prec., mutual ex.
1.2 Example

\[ H = \text{lcm}(4, 12) = 12 \]
\[ U = \frac{1}{4} + \frac{9}{12} = 1 \]

[LM80]

[GD99]

[CG04]
### 1.3 Comparison

- U=80% and 90%, UUnifast with random periods, \( O_i \) uniform in \([0..T_i]\)
  - But LCM limited to 50 million time units => bias (more harmonic periods for higher number of tasks)

- 10000 configurations/point

<table>
<thead>
<tr>
<th># tasks</th>
<th>Simu = H</th>
<th>( s_n/\text{exact} )</th>
<th>( (s_n-\text{exact})/T_{\max} )</th>
<th>LM80/\text{exact}</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>51%</td>
<td>102.3%</td>
<td>72% (200%)</td>
<td>200.6%</td>
</tr>
<tr>
<td>5</td>
<td>43%</td>
<td>100.1%</td>
<td>91% (300%)</td>
<td>200%</td>
</tr>
<tr>
<td>7</td>
<td>36%</td>
<td>100%</td>
<td>111% (351%)</td>
<td>200%</td>
</tr>
<tr>
<td>10</td>
<td>34%</td>
<td>100%</td>
<td>109% (465%)</td>
<td>200%</td>
</tr>
</tbody>
</table>

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</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>39%</td>
<td>102.3%</td>
<td>62% (200%)</td>
<td>200.1%</td>
</tr>
<tr>
<td>5</td>
<td>30%</td>
<td>100.1%</td>
<td>80% (271%)</td>
<td>200%</td>
</tr>
<tr>
<td>7</td>
<td>28%</td>
<td>100%</td>
<td>103% (382%)</td>
<td>200%</td>
</tr>
<tr>
<td>10</td>
<td>35%</td>
<td>100,1%</td>
<td>109% (432%)</td>
<td>200%</td>
</tr>
</tbody>
</table>
[LM80] is **twice** too long in average!
- Twice means more than H too long!

[GD99] has an average overestimation of $T_{\text{max}}$
- Max observed over 80000 random systems
  $4.65*T_{\text{max}}$ (very small compared to H)

30-50% systems require only [0,H)
2. MULTIPROCESSOR SYSTEMS
2.1 Counter-intuitive example

<table>
<thead>
<tr>
<th>Task</th>
<th>O_i</th>
<th>C_i</th>
<th>D_i</th>
<th>T_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ₁</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>τ₂</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>τ₃</td>
<td>0</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

- Synchronous system, arbitrary deadlines
- Longest Remaining Processing Time First
- Simulation [0,4)=[0,H)
2.1 Counter-intuitive example / contd

- EDF, same system
- Simulation [0,3H)
2.1 Counter-intuitive example /contd

- DM, same example
  - Infinite transient phase

Missed deadline

\( \tau_1 \)
\( \tau_2 \)
\( \tau_3 \)
\( \Pi_1 \) idle
\( \Pi_2 \) idle

\( \infty \) transient phase
2.2 State of the art

- Most of the time not comparable
  - « Best duration » = min applicable bounds

[GD97]FJP: [0, $s_n + H$)
[GD99]FJP: [0, $O_{\text{max}} + 2H$]
[LM80]FTP: [0, $O_{\text{max}} + 2H$]

[CG04]Conv: $H(1 - U)$ idle slots
[CG07]gFTP: [0, $\delta_n + H$]
[CG06][CG11] gFTP: [0, $s_n + H$)

[BB12]Any: [0, $H \prod (C_i + 1)$]
[GGC13]Any: [0, $H \prod (O_i + D_i - T_i)_0 + 1$]
[NYG13]Any: [0, $s_n + H$]

Arbitrary deadlines
Dependent tasks

Uniprocessor
Identical multiprocessor
Uniform & Unrelated multiprocessor
3. Open problems

- **Uniprocessor**
  - Exact start of the steady phase
  - Exact bound for arbitrary deadlines
  - WCET « increasing » to account for preemptions

- **Multiprocessor**
  - Study multiprocessor sustainability
  - Better understanding of cyclic behavior in multiprocessor systems
  - Complement of what we know
    - Better bounds (exact multiprocessor bound?)
    - Unrelated/uniform (other than independent, gFTP)
References